doi: 10.1255/tosf.73

The overall measurement error—TOS and uncertainty budget in metal accounting

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Metal accounting is one of the main tools for financial and technical management of metal production industry. It is based on measurements and has to manage the uncertainty inherent to the measurement process. The uncertainty in the metal accounting generates financial risk. The accuracy of the metal accounting results is directly linked to the accuracy of the material balance and then to the accuracy of the mass and content measurements. Estimate the overall measurement error, through its probability distribution or its first and second moments (mean and variance), can contribute to the enterprise decision making.

The overall measurement error can be calculated and analysed by establishing the uncertainty budget. If this approach has been mainly introduced to calculate the analytical error (cf. ISO GUM), it has to take into account the sampling procedure. Even though it is not explicitly named "uncertainty budget", the same approach is proposed in the Pierre Gy's Theory of Sampling (TOS), where the various components of the overall error are well identified and described with their properties and their relative weights.

The present paper proposes a methodology to build such uncertainty budgets in the frame of the implementation of a metal accounting system. It can be applied to an existing measurement system, analysing the results in order to find some ways for improving the measurement accuracy. In addition, it can be used to define a new measurement procedure with an objective of accuracy. Various real examples illustrate both applications.

Introduction

If "Metal accounting is the estimation of (saleable) metal produced by the mine and carried in subsequent process streams over a defined period of time"[1], it has become widely used to quantify the performances of production plants (metal recovery, losses, environmental impact) and to establish an accurate estimation for the metal inventory (stock taking and work in progress estimation). A large discrepancy between the estimated and actual inventory can have significant financial consequences. Similarly, poor estimation of metal recovery and losses can hide process issues and give inappropriate production planning. This is why "metal accounting provides interface between technical and financial performance measurement"[1]. These two cultures have two very different points of view and have difficulties to conciliate them. The main topic of disagreement is the uncertainty of measurement which implies uncertainty in the estimation of production and inventories.

The measurement uncertainty and the methods of reducing it have been largely discussed in many papers[2,3,4,5,6]. The objective of the current paper is to propose a method to be able to quantify the uncertainty with the establishment of the uncertainty budget of any measurement useable for metal accounting. An audit of the measurement system has to take place in order to examine the current situation, collect all information necessary for uncertainty budget and make recommendations for measurement accuracy improvements.

Metal accounting implementation

Metal accounting is a component of the general enterprise accounting[7,8]. It constitutes a powerful tool to manage metal producing companies at their various stages: mine and mill, concentrator, smelter or hydrometallurgical plant, refinery, or a combination of these stages. It is the bridge between the technical and the financial point of views of the process. The process data generated to manage the production performances are used to valuate the products and stocks into financial data.

The main objective of a metallurgical accounting system is to help the company in managing process data to generate a material balance in order to obtain a metal accounting report. The secondary objective is to use the material balance to accurately calculate the process performances and help the process manager in optimising it. The metal accounting is generally established for a period of production. This period can be defined by a regular time period or by the period of production of a material batch. In accordance with the financial and accounting rules, the regular time period is generally a month.

In the life time of a company we can consider three life cycle levels for metal accounting[9]:

- Metal accounting system life cycle: this begins with the decision to implement the metal accounting system in a company and finishes with the decision to end it.
- Production evolution life cycle: this regards the adjustments of the metal accounting system due to production evolutions such as a process change, a new production unit, or new products.
- Metal accounting life cycle: this groups the periodical tasks to obtain a regular metal accounting report.

From the moment a company decides to implement a metal accounting system to the time the system reaches completion, three periods can be identified. The "implementation" groups all tasks to obtain an operational and efficient metal accounting system. The "production" groups all tasks to regularly generate metal accounting reports and update the system according to notable evolutions. The "closing" groups all tasks to finalize the last metal accounting taking into account the plant dismantlement.

The implementation of a metal accounting system is a company project mobilising all staff: general management, financial, accounting, production, laboratory, metrology, information technology, purchasing, sales staff… Depending on the initial level of development of the company many tasks have to be taken into consideration[9]. The ones concerning the present paper are: a review of the existing measurement system; the design and implementation of necessary additional measurements; the establishment of the measurement uncertainty budget[6] involving the identification and implementation of some improvements; the standardisation of the measurement system.

Measurements at the basis of metal accounting

The metallurgical accounting is based on the calculation of the material balance of the considered system. This calculation necessitates raw data, such as masses, moisture contents or assays, which are obtained by measurements. As the measurement is a random process, it is subject to uncertainty which can be quantified with its associated "measurement error"[2]. It concerns also the measurements of mass[3], moisture or metal content, percentage of solids or density… These last measurements generally necessitate sampling which is the main source of uncertainty[4,5]. All efforts have to be done to obtain correct sampling and measurement to avoid any bias. This bias would produce discrepancies between metal accounting and real production with the risk of unacceptable financial consequences. Nevertheless, the variance of the overall measurement error cannot be avoided and its calculation necessitates the establishment of its uncertainty budget[6].

The quantity of material managed during the considered period of metal accounting is generally given by the sum of many mass measurements such as truck loads or production weights per shift. Similarly, the mean moisture or metal contents are calculated by the weighted average of the contents of many samples. The aggregation of this raw data gives the "basis data" which is the sum of the total masses or the average contents of the material during the accounting period. A measurement error can then be attached to the basis data using the error propagation calculation rules[10].

Measurement error and data reconciliation

Due to the measurement uncertainty, the basis data are incoherent regarding the material conservation laws[11,12]. The incoherence can be observed when there is data redundancy: when there is more data than the required minimum to calculate the material balance. The objective of data reconciliation by material balance is to find a set of estimates for the measured values which are as close as possible to the measurements and verify the material conservation laws. Sometimes, balancing behaviour reveals non-stationary processes or bad accuracy estimation. The information redundancy allows delivering coherent estimators more accurate than the initial measurements[13,14]. This approach allows for the detection of aberrant values and to reduce error due to sampling and measurement.

Overall measurement error

The relative measurement error is defined as the difference between the value of a parameter obtained by a measurement protocol and the true value which is, by definition, unknown, the whole divided by the true value. Due to the natural variability occurring in any measurement protocol, the measurement error is a random variable following a probability law which can be obtained using different approaches. For the statistical approach, the same measurement is performed a large number of times and statistics are done on the set of results. This approach, referring to evaluation of type A, is called a posteriori as it is necessary to do the measurements to be able to evaluate the probability law. The probabilistic approach, evaluation of type B, is called a priori because it is based on theories such as the sampling theory. A combination of these two approaches can be used to evaluate the overall measurement error. The moments of the probability distribution are used to characterise the measurement error. The first moment, the mean, gives an evaluation of the bias, a systematic deviation between the measurements and the true value. It measures the accuracy of the measurement. The second moment, the variance, quantifies the reproducibility (or precision) of the measurement.

Components of the measurement error

The overall measurement error (OE) includes a lot of components which can be divided, following the Pierre Gy's classification[15, 16, 17], into two main components: the total sampling error (TE) and the analytical error (AE).

The analysis error is due to the imperfection of the protocols and devices used for analytical operations[19]. When concerning assaying or moisture content, the analysis is performed on the sample obtained from the last sampling stage, which is generally taken in the laboratory. The evaluation of the analysis error needs the decomposition of the protocols and procedures to find all sources of error. Calculation rules and metrological approach are used to calculate the total analysis error. Another approach, mainly used in QAQC procedures, is based on the variance analysis of a large number of performed measurements[18].

The total sampling error has to take into account the succession of particle size and bulk reductions. It is then the sum of the total sampling errors at each stage (TEn). The sample preparation operations generate the increment preparation error (IPE) due to contamination, loss, chemical or physical alteration, unintentional or intentional mistakes. The operation of taking a small amount of material in a lot in order to obtain a sample generates: the fundamental sampling error (FSE), the grouping and segregation error (GSE), all together called short-range process integration error (PIE_1), result from the heterogeneity of constitution, while the long-range (PIE_2) and periodic (PIE_3) process integration errors, and the increment weighing error (IWE) comes from the heterogeneity of distribution in the space or in the time. The increment delimitation error (IDE) and the increment extraction error (IEE) constitute the materialisation error.

Uncertainty budget

The evaluation of the overall measurement error necessitates listing all the sources of error along the entire process, from the original lot, subject to the measurement, to the use of the analytical results. The inventory of the sources of errors is obtained from a preliminary diagnostic phase of the plant measurement system. This phase has a double objective: calculate the variance of the overall measurement error and improve the measurement process, everywhere it is possible, in order to reduce this error in terms of bias and variance.

The uncertainty budget lists all the components of the overall measurement error with their respective weights. The analysis of the repartition of the components allows focussing on the improvement of the main components. The establishment of such a list necessitates an a priori approach of type B. Indeed, it is very difficult to extract the error components from the variance analysis of a large number of measurements in the frame of an a posteriori approach of type A. Nevertheless, some components can be obtained from such type A approach such as the device repeatability or, concerning sampling, the process integration errors obtained from chronostatistics[15,16].

Audit of the measurement system

The term “Measurement System” refers to all aspects of the measurements:

- All pieces of equipment used for measurements including sample taking and preparation, and laboratory;
- Their documentation: manuals, maintenance log-sheets, calibration log-sheets and certificates, inventory;
- The measurement procedures including sample taking and preparation, analysis;
- The measurement results management and storage;
- The Quality Assurance / Quality Control (QA/QC) documentation: procedures, reports;
- The uncertainty budget for all relevant measurements;
- Information and data repository.

The first step of the audit is to examine the current situation of the measurement system. It starts with an inventory of all the measurements required for metal accounting. This list is confronted with the inventory of the currently performed measurements. A special attention has to be paid at this level. Indeed, the definition of the material flow diagram[9] (including material movements and stocks which are accounted) and its level of details is commonly conducted by the availability of measurements, while common sense would dictate the contrary: define the material flow diagram with the objective of accurate metal accounting and then locate, design and implement the measurements. The comparison between the inventories of the expected and actual measurements gives a first idea of the “cost” for the measurement system upgrading.

The already performed measurements are then analysed in details one by one. An on-site visit is absolutely necessary to observe the measurement process in operation: true location of the measurement or of the sampling point, material subject to measurement, operating conditions of the equipment, operator practice, operating environment… All the documentation concerning equipment (such as user’s guide, technical sheets, and maintenance and calibration log-sheets), procedures (for sampling taking and preparation, analysis and safety rules), QA/QC (procedures and reports) and material are collected. Results of already performed measurements have to be collected from databases (historians) or log-sheets for subsequent statistical analysis.

The technical documentation of the equipment allows to list its inherent sources of error and to collect the quantitative values used to estimate their components in form of variances (such as readability or temperature sensitivity). The procedures give the detailed description of the measurement process with all its steps and the sources of error arising at each stage. The list of items to account in the uncertainty budget is deducted from these both kinds of documentation. If documentation is missing, operator interview is absolutely required. Even though the documentation is available, such interview is always rewarding as there is always a gap between the documentation and the real practice.

Material characterisation for heterogeneity model

The theory of sampling gives guidance how to calculate the fundamental sampling error starting from the description of the heterogeneity of the material regarding the parameter to measure (moisture content, assay, slurry density). A detailed description of the heterogeneity can be deducted from various sources of information: mineralogical studies including quantitative mineralogy using image analysis, size and density distribution analysis, processing test results and process data. Such a model of heterogeneity has to be developed for each stage of the sampling plan. Indeed, the material being ground before sub-sampling, the heterogeneity changes in terms of size distribution and mineral liberation. Generally, most of the required information are available in the collected documentation. If there are missing data, specific experiments can be conducted to refine the material characterisation.

The variographic analysis is the better way to estimate the components of the process integration error (PIE) and mainly the ones associated to the distribution heterogeneity[15,16,20]. Such studies are rarely available before the audit. Sometimes, the historical data are sufficient to have a first idea of the process variability. But the required operating conditions to conduct such a study are not the one of the routine measurements. It is why it has to be performed for the more relevant sampling points, that is to say where the benefit will cover the cost.

A multi-disciplinary approach is absolutely necessary during this task. Indeed, the heterogeneity model is built by inference from a great diversity of information sources. In addition, it is generally necessary to do, and justify, some realistic assumptions.

Audit report

The main part of the audit report concerns the uncertainty budget. It allows to associate a quantitative error to each measurement, what is the basis of data reconciliation. The uncertainty budget highlights the main components of the overall error pointing out the possible improvements. Recommendations can then be done at the light of these results.

Conclusion

A metallurgical accounting system has to conciliate two points of view: the technical point of view for which the material balance is the product of a statistical approach of the reality and the financial point of view for which the metal balance refers to an exact and coherent economic value in the accounting system. Nevertheless, the material balance is based on measurements which are random processes. The measurement error has to be considered when corrections take place during the data reconciliation process. If the data reconciliation is based on a statistical coherent material balance, the obtained estimated values are the more probable ones.

In the implementation of a Metal Accounting system, the initial diagnostic of the existing measurement system has to be carefully conducted to have a good quantification of the overall measurement error. This error is directly used by the data reconciliation system and gives the accuracy of the key point indicators. In addition, the analysis of the uncertainty budget of the overall error indicates the main components on which the efforts of improvement have to be done.

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